339: Prove a spin-statistics theorem (e.g. in terms of resolving vs. non-resolving branch pairs) for Wolfram model multiway systems, and explore connections to the Dirac equation

Investigate possible models for particle statistics in Wolfram model systems (e.g. interpreting Bose-Einstein statistics as corresponding to localized topological obstructions whose branch pairs resolve, and Fermi-Dirac statistics as corresponding to those whose branch pairs do not), and explore connections to angular momentum (e.g. by examining net fluxes of causal edges through rotating timelike hypersurfaces), and its possible quantization due to homotopical properties of the multiway causal graph. Use mathematical analysis to deduce relationships to the Dirac equation, parastatistics (e.g. in terms of conditionally-resolvent or probabilistically-resolvent branch pairs) and the spin-statistics theorem. Use explicit computational experiments to reproduce relevant correlations such as exclusion principles and the Hanbury Brown and Twiss effect.

A brief timeline and Goals:

Grand Goal: PhD in theoretical condensed matter:

Heard of Xerxes about Tensor Networks and Multiway system graphs, I might want to do something about it in the future, develop some fundamental rules using the wolfram model?

Now: start to learn and explore more on multiway systems, hypergraphs and quantum mechanics, learn about some ideas / make some connections between ladder / string operators with the wolfram model ; STILL DIGESTING

Project reason: interested in quantum mechanics, interesting approach using the wolfram model, want to help develop the wolfram model more in terms of quantum mechanics, whilst learning more

Defining the terms:

spin-statistics theorem

resolving vs. non-resolving branch pairs model

multiway systems

Dirac equation: Relativistic Quantum Mechanics

Bose-Einstein statistics: for Bosons (integer spins)

localized topological obstructions: some action hindered/not supported/not allowed due to topology

Fermi-Dirac statistics: for Fermions (half integer spins)

angular momentum: intrinsic magnetic moment for particles

net fluxes of causal edges through rotating timelike hypersurfaces

its possible quantization due to homotopical properties of the multiway causal graph.

parastatistics (e.g. in terms of conditionally-resolvent or probabilistically-resolvent branch pairs)

exclusion principles: No fermions can exist in the same state |0> or |1>; bosons 🡪 |n>, n is a collection of integers

Hanbury Brown and Twiss (HBT) effect:

From the Wolfram Glossary: <https://arxiv.org/pdf/2010.02752.pdf> pg 82

• Branchial Graph: The graph whose vertex set is the set of states in a particular layer (or slice) of the multiway evolution graph, and in which states are connected by directed edges if and only if they share a common ancestor in the evolution graph. Otherwise known as a branchlike hypersurface, by analogy to spacelike hypersurfaces in causal networks. Branchial graphs are used to represent instantaneous superpositions between pure states.

https://www.wolframphysics.org/technical-introduction/the-updating-process-for-string-substitution-systems/the-concept-of-branchial-graphs/

• Branchial Space: The spatial structure defined by a branchial graph, much like how physical space is the spatial structure defined by a hypergraph. In this way, branchial space has the same relationship to the multiway evolution graph as physical space has to an ordinary causal network. The default metric on branchial space (i.e. the Fubini-Study metric) is defined in such a way that entangled states are nearby.

• Branchtime: The spatial structure defined by a multiway evolution graph, i.e. the time-extended version of branchial space. In this way, branchtime has the same relationship to the multiway evolution graph as spacetime has to an ordinary causal network.

• Causal Network: The network of causal relationships between updating events. Specifically, the vertex set of the causal network is the set of updating events, and a directed edge exists between events A and B if and only if the input for event B makes use of hyperedges that were produced by the output of event A, such that event B could not be applied unless event A had previously been applied. The flux of edges through particular hypersurfaces in the causal network is related to certain projections of the energy-momentum tensor.

• Causal Invariance: A property of multiway systems whereby all possible evolution paths yield causal networks that are (eventually) isomorphic as directed acyclic graphs. Since the notion of confluence 82 in the theory of abstract rewriting systems is a necessary (though not sufficient) condition for causal invariance, it follows that whenever causal invariance exists, every branch in the multiway evolution graph must eventually merge. For the particular case of a terminating (strongly normalizing) rewriting system, causal invariance therefore implies that all evolution paths yield the same eventual state. For a causal network representing the causal structure of a Lorentzian manifold, causal invariance implies Lorentz symmetry.

• Completion: An additional rule or collection of rules introduced into a multiway system that brings it closer to causal invariance (some multiway systems can be made causal invariant by adding only a finite number of completions). Completions, specifically Knuth-Bendix completions, are commonly used in automated theorem-proving algorithms, as a means of forcing confluence within equational rewriting systems. Knuth-Bendix completions also constitute a conjectural approach to representing the process of projective quantum measurement within the framework of the Wolfram model, by allowing one to “collapse” superpositions of states in branchial space.

• Foliation: A method for defining a universal time function over the vertices of a directed acyclic graph (i.e a function mapping vertices to integers), in such a way that the level sets of that function, known as slices, cover the entire graph without intersecting. Foliations of a causal network yield successive configurations of hypergraphs, representing spacelike hypersurfaces. Foliations of a multiway evolution graph yield successive configurations of branchial graphs, representing instantaneous superpositions between pure states (branchlike hypersurfaces).

• Hypergraph: The basic structure used for representing space in the Wolfram model. Hypergraphs are generalizations of ordinary graphs in which hyperedges can connect any arbitrary non-empty subset of vertices (such that ordinary graphs correspond to the special case in which all hyperedges are of arity 2). In this way, a hypergraph can be represented purely formally as a collection of abstract relations between elements.

• Multiway Evolution Causal Graph: A composition of a multiway evolution graph and a causal network, i.e. a directed acyclic graph containing both global states and updating events, and in which there exist two fundamentally different kinds of edges - evolution edges, which connect different states in the multiway evolution, and causal edges, which show the causal relationships between the updating events. In this way, the multiway evolution graph contains information regarding the causal relationships not only on a single branch of multiway evolution (as in a standard causal network), 83 but also between different branches of multiway evolution. In a causal invariant system, the multiway evolution causal graph can be effectively factored into many isomorphic spacetime causal networks.

• Multiway Evolution Graph: A directed acyclic graph whose vertex set is the set of possible states of a given multiway system, and a directed edge exists between states A and B if and only if there exists an updating event (i.e. a rule application) that transforms state A to state B. The limiting behavior of geodesics in the multiway evolution graph is governed by a path integral.

• Multiway System: An abstract rewriting system that has been enriched with additional causal structure between rewriting events (see Multiway Evolution Graph and Multiway Evolution Causal Graph for further details).

• Observer: Any ordered sequence of non-intersecting level surfaces of a universal time function, defined over a directed acyclic graph. In the case of a causal network, this corresponds to a foliation of spacetime (and therefore to a relativistic observer, embedded in a particular reference frame). In the case of a multiway evolution graph, this corresponds to a foliation of branchtime (and therefore to a quantum observer).

• Rulial Multiway System: The multiway system constructed by applying all possible rules of a given class (e.g. hypergraph tansformation rules, string substitution rules, Turing machine rules, etc.) to states of a given system. For instance, the rulial multiway system for Turing machines is obtained by allowing all possible non-deterministic transitions between Turing machine states. See Rulial Space for further details.

• Rulial Space: The space defined by allowing all possible rules of a given class (e.g. hypergraph transformation rules, string substitution rules, Turing machine rules, etc.) to be followed between states of a system. In other words, it is the spatial structure associated with the evolution of a rulial multiway system. For instance, the rulial space for Turing machines is obtained by allowing all possible non-deterministic transitions between Turing machine states. Rulial space can be formulated as a category of rewriting rules, and it functorially acquires the properties of an adhesive category. Consequently, the monoidal structure of both multiway evolution graphs and branchial graphs (which are obtained by foliations of multiway evolution graphs) is inherited from the composition of rules in rulial space.

• Spacelike Hypersurface: A subset of spacetime that contains only spacelike-separated events, i.e. 84 events that form antichains in the associated causal network, such that a time-ordered sequence of spacelike hypersurfaces defines a foliation of spacetime. The events on a spacelike hypersurface may be considered by a relativistic observer to be “simultaneous”. In the Wolfram model, the flux of causal edges through spacelike hypersurfaces is associated with energy.

• Timelike Hypersurface: A subset of spacetime that contains only timelike-separated events, i.e. events that are connected by edges in the associated causal network, and which are therefore orthogonal to spacelike hypersurfaces. In the Wolfram model, the flux of causal edges through timelike hypersurfaces is associated with momentum.

• Updating Event: A single application of a rule (i.e. an application of the rewrite relation →) in a multiway system. Updating events form the set of vertices in the causal network and the set of edges in the multiway evolution graph.

<https://writings.stephenwolfram.com/2021/05/the-problem-of-distributed-consensus/>

https://www.wolfram.com/events/distributed-consensus/agenda/